Maximum Marks: 200 Time: 45 Min.

General Instructions:

- (i) Section A will have 15 questions covering both i.e., Mathematics/Applied Mathematics which will be compulsory for all candidates.
- (ii) Section B1 will have 35 questions from Mathematics out of which 25 questions need to be attempted. Section B2 will have 35 questions purely from Applied Mathematics out of which 25 questions will be attempted.
- (iii) Correct answer or the most appropriate answer: Five marks (+ 5)
- (iv) Any incorrect option marked will be given minus one mark (-1).
- (v) Unanswered/Marked for Review will be given no mark (0).
- (vi) If more than one option is found to be correct then Five marks (+5) will be awarded to only those who have marked any of the correct options.
- (vii) If all options are found to be correct then Five marks (+5) will be awarded to all those who have attempted the question.
- (viii) If none of the options is found correct or a Question is found to be wrong or a Question is dropped then all candidates who have appeared will be given five marks (+5).
- Calculator / any electronic gadgets are not permitted.

Section - A

Mathematics/Applied Mathematics

1. If
$$f(x) = \begin{cases} mx + 1 & \text{if } x \le \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$$
 is continuous at

$$x = \frac{\pi}{2}$$
 then

(1)
$$m = 1, n = 0$$

(1)
$$m = 1, n = 0$$
 (2) $m = \frac{n\pi}{2} + 1$

(3)
$$n = \frac{m\pi}{2}$$
 (4) $m = n = \frac{\pi}{2}$

(4)
$$m = n = \frac{\pi}{2}$$

2. If
$$\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$$
, then the value of x is

- **(1)** 3
- **(2)** 0
- (3) 1
- **(4)** 1

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

- (A) Both A and R are true and R is the correct explanation of A
- (B) Both A and R are true but R is NOT the correct explanation of A
- **(C)** A is true but R is false
- (D) A is false and R is True

3. Assertion (A):
$$\int_{0}^{\pi/2} \frac{\cos x}{\sin x + \cos x} \, dx = \frac{\pi}{4}$$

Reason (R):
$$\int_{0}^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{4}$$

4. Assertion (A):
$$\frac{d}{dx} \left[\int_{0}^{x^2} \frac{dt}{t^2 + 4} \right] = \frac{2x}{x^4 + 4}$$

Reason (R):
$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$$

- **5.** Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if \bar{a} is congruent to $b \forall a, b \in T$. Then R is
 - (1) reflexive but not transitive
 - (2) transitive but not symmetric
 - (3) equivalence relation
 - (4) None of these
- **6.** The area of the region bounded by the *y*-axis, $y = \cos x$ and $y = \sin x$, $0 \le x \le \pi/2$ is

 - (1) $\sqrt{2}$ sq. units (2) $(\sqrt{2} + 1)$ sq. units
 - (3) $(\sqrt{2} 1)$ sq. units (4) $(2\sqrt{2} 1)$ sq. units
- **7.** A ladder, 5 metre long, standing on a horizontal floor, leans against a vertical wall. If the top of the ladder slides downwards at the rate of 10 cm/sec, then the rate at which the angle between the floor

and the ladder is decreasing when lower end of ladder is 2 metre from the wall is:

- (1) $\frac{1}{10}$ radian/sec
- (2) $\frac{1}{20}$ radian/sec
- (3) 20 radian/sec
- (4) 10 radian/sec
- **8.** If A and B are two events such that $P(A) \neq 0$ and P(B|A) = 1, then
 - (1) $A \subset B$
- (2) $B \subset A$
- (3) $B = \varphi$
- **9.** The value of $\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$ is
 - (1) $\frac{10}{10}$

- **10.** If P(A|B) > P(A), then which of the following is correct:
 - **(1)** P(B|A) < P(B)
- (2) $P(A \cap B) < P(A).P(B)$
- (3) P(B|A) > P(B)
- **(4)** P(B|A) = P(B)
- **11.** The equation of normal to the curve $3x^2 y^2 = 8$ which is parallel to the line x + 3y = 8 is
 - (1) 3x y = 8
- (2) 3x + y + 8 = 0
- (3) $x + 3y \pm 8 = 0$
- (4) x + 3y = 0
- **12.** Consider the non-empty set consisting of children

- in a family and a relation R defined as aRb if a is brother of b. Then R is
- (1) symmetric but not transitive
- (2) transitive but not symmetric
- (3) neither symmetric nor transitive
- (4) both symmetric and transitive
- **13.** The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20), (60, 0). The objective function is

Compare the quantity in Column A and Column B

Column A	Column B
Maximum of Z	325

- (1) The quantity in column A is greater.
- (2) The quantity in column *B* is greater.
- (3) The two quantities are equal.
- (4) The relationship cannot be determined on the basis of the information supplied.
- **14.** The value of $\int_{-\pi/2}^{\pi/2} (x^3 + x \cos x + \tan^5 x + 1) dx$ is
 - **(1)** 0
- (3) π
- **(4)** 1
- **15.** Distance of the point (α, β, γ) from *y*-axis is
- (3) $|\beta| + |\gamma|$
- (4) $\sqrt{\alpha^2 + \gamma^2}$

Mathematics

- **16.** The value of $\sin^{-1} \left[\cos \left(\frac{33\pi}{5} \right) \right]$ is
 - (1) $\frac{3\pi}{5}$
- (2) $\frac{-7\pi}{5}$
- **17.** If $y = \log_e \left(\frac{x^2}{e^2}\right)$, then $\frac{d^2y}{dx^2}$ equals
 - (1) $-\frac{1}{x}$
- (2) $-\frac{1}{x^2}$
- (4) $-\frac{2}{3}$
- **18.** If the curve $ay + x^2 = 7$ and $x^3 = y$, cut orthogonally at (1, 1), then the value of a is :
 - **(1)** 1
- **(2)** 0
- (3) -6
- **19.** Let $A = \begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 50 & 40 \\ 2 & 3 \end{bmatrix}$, then |AB|
 - is equal to
 - **(1)** 460
- **(2)** 2000
- (3) 3000
- (4) 7000

- **20.** If A and B are any two events such that P(A) + P(B) - P(A and B) = P(A), then
 - (1) P(B|A) = 1
- (2) P(A|B) = 1
- (3) P(B|A) = 0
- **(4)** P(B|A) = 0
- **21.** The sine of the angle between the straight line

 $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane 2x - 2y + z = 5 is

- (3) $\frac{2\sqrt{3}}{5}$ (4) $\frac{\sqrt{2}}{10}$
- **22.** The area of the region bounded by the curve $x^2 = 4y$ and the straight-line x = 4y - 2 is

 - (1) $\frac{3}{8}$ sq. units (2) $\frac{5}{8}$ sq. units
 - (3) $\frac{7}{8}$ sq. units (4) $\frac{9}{8}$ sq. units

- **23.** The value of determinant $\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \end{vmatrix}$ is
 - (1) $a^3 + b^3 + c^3$
- (3) $a^3 + b^3 + c^3 3abc$
- (4) None of these
- **24.** In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is
 - (1) 10^{-1}
- (2) $\left(\frac{1}{2}\right)^5$
- (3) $\left(\frac{9}{10}\right)^5$
- **25.** The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$$
 is

- **(1)** 1
- (3) 3
- (4) not defined
- **26.** The set of points where the function f given by $f(x) = |2x - 1| \sin x$ is differentiable is
- (2) $R \left\{ \frac{1}{2} \right\}$
- (3) $(0, \infty)$
- (4) none of these
- **27.** The equation of tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses x-axis is :
 - (1) x + 5y = 2
- **(2)** x 5y = 2
- (3) 5x y = 2
- (4) 5y + x = 2
- **28.** If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}, \text{ then } R \text{ is }$
 - (1) reflexive
- (2) transitive
- (3) symmetric
- (4) None of these
- **29.** The distance of the plane $\vec{r} \cdot \left(\frac{2}{7} \hat{i} + \frac{3}{7} \hat{j} \frac{6}{7} \hat{k} \right) = 1$
 - from the origin is
 - **(1)** 1
- (2) 7
- (4) None of these
- **30.** The value of $\tan \left| \frac{1}{2} \cos^{-1} \left(\frac{\sqrt{5}}{3} \right) \right|$ is
 - (1) $\frac{3+\sqrt{5}}{2}$
- (2) $\frac{3-\sqrt{5}}{2}$

 - (3) $\frac{-3+\sqrt{5}}{2}$ (4) $\frac{-3-\sqrt{5}}{2}$
- **31.** $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ is equal to
 - $(1) \frac{-1}{\sin x + \cos x} + C$
 - (2) $\log |\sin x + \cos x| + C$
 - (3) $\log |\sin x \cos x| + C$
 - $(4) \ \frac{1}{(\sin x + \cos x)^2}$

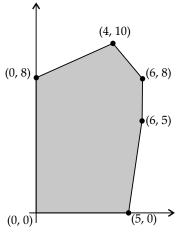
- **32.** The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ are
 - **(1)** 1
- **(2)** 2
- (3) 3
- (4) 5
- **33.** The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is
 - **(1)** 1
- (3) 5
- **34.** If $y = e^{-x} (A\cos x + B\sin x)$, then y is a solution of

 - (1) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$ (2) $\frac{d^2y}{dx^2} 2\frac{dy}{dx} + 2y = 0$
 - (3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ (4) $\frac{d^2y}{dx^2} + 2y = 0$
- **35.** The probability of obtaining an even prime number on each die, when a pair of dice is rolled is
 - **(1)** 0

- **36.** Let \vec{a} , \vec{b} , and \vec{c} be three unit vectors, out of which vectors b and c are non-parallel. If α and β are the angles which vector a makes with vectors b and crespectively and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}}{2}$, then $|\vec{\alpha} - \vec{\beta}|$ is equal to:
 - (1) 30°
- $(2) 90^{\circ}$
- $(3) 60^{\circ}$
- $(4) 45^{\circ}$
- **37.** The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line passing through the point (2, 3, -4) and parallel to the vector $6\hat{i} + 3\hat{j} - 4\hat{k}$ is:
 - **(1)** 7
- (2) $4\sqrt{3}$
- (3) $2\sqrt{13}$
- **38.** If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to
 - (1) $\frac{4x^3}{1-x^4}$ (2) $\frac{-4x}{1-x^4}$
- **39.** Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} \vec{b}$ has the magnitude 12 then one such vector is:

 - (1) $4(2\hat{i}-2\hat{j}-\hat{k})$ (2) $4(2\hat{i}-2\hat{j}+\hat{k})$
 - (3) $4(2\hat{i} + 2\hat{j} + \hat{k})$
- (4) $4(2\hat{i} + 2\hat{j} \hat{k})$
- **40.** The sum of the distinct real values of μ , for which the vectors, $\mu \hat{i} + \hat{j} + \hat{k}$, $\hat{i} + \mu \hat{j} + \hat{k}$, $\hat{i} + \hat{j} + \mu \hat{k}$ are coplanar, is:
 - **(1)** –1
- **(2)** 0
- **(3)** 1
- (4) 2

41. The feasible solution for a LPP is shown in given figure. Let Z = 3x - 4y be the objective function Minimum of Z occurs at



- (1) (0,0)
- **(2)** (0, 8)
- (3) (5,0)
- **(4)** (4, 10)
- **42.** Let $\vec{\alpha} = (\lambda 2)a + b$ and $\beta = (4\lambda 2)a + 3b$ be two given vectors where a and b are non collinear. The value of λ for which vectors $\vec{\alpha}$ and β are collinear, is:
 - (1) -4
- **(2)** –3
- (3) 4
- **(4)** 3
- **43.** Let $f(x) = |\sin x|$, then
 - **(1)** *f* is everywhere differentiable
 - **(2)** *f* is everywhere continuous differentiable at $x = n\pi$, $n \in \mathbb{Z}$.
 - (3) *f* is everywhere continuous but not differentiable at $x = (2n + 1) \frac{\pi}{2}, n \in Z$.
 - (4) none of these
- **44.** The integrating factor of differential equation

$$\cos x \frac{dy}{dx} + y \sin x = 1 \text{ is}$$

- (1) $\cos x$
- (2) tan x
- (3) $\sec x$
- (4) $\sin x$
- **45.** If the direction cosines of a line are k, k, k, then
 - **(1)** k > 0
- (2) 0 < k < 1
- (3) k = 1
- (4) $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

- **46.** The area of a triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 sq. units. Then, the value of k will be
 - (1)9
- **(2)** 3
- (3) -9
- **(4)** 6
- **47.** Refer to Q. 41 maximum of Z occurs at
 - (1) (5,0)
- (2) (6,5)
- **(3)** (6, 8)
- **(4)** (4, 10)

Read the following text and answer the following questions on the basis of the same:

On her birthday, Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However, if there were 16 children more, everyone would have got ₹10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in ₹).



- **48.** The equations in terms of x and y are
 - (1) 5x 4y = 40
- (2) 5x 4y = 40
- 5x 8y = -80
- 5x 8y = 80
- (3) 5x 4y = 40
- **(4)** 5x + 4y = 40
- 5x + 8y = -80
- 5x 8y = -80
- 49. Which of the following matrix equations represent the information given above?
- **50.** The number of children who were given some money by Seema, is
 - **(1)** 30
- **(2)** 40
- (3) 23
- **(4)** 32

Section - B2

Applied Mathematics

- **16.** If $a \equiv b \pmod{n}$, then
 - (1) $n \mid a$ and $n \mid b$
- (2) $n \mid b$ only
- (3) n | (a b)
- (4) None of these
- **17.** Evaluate: $(9 + 23) \mod 12 = \dots$
 - **(1)** 2
- **(2)** 8
- (3) 32
- **(4)** 12

- **18.** If B > A, then which expression will have the highest value, given that A and B are positive integers.
 - (1) A B
- (2) $A \times B$
- (3) A + B
- (4) Can't say
- **19.** Tea worth ₹ 126 per kg and ₹ 135 per kg are mixed with a third variety in the ratio 1:1:2. If the mixture

per kg will be:

- **(1)** ₹ 169.50
- **(2)** ₹ 170
- (3) ₹175.50
- (4) ₹180
- **20.** $A = [a_{if}]_{m \times n}$ is a square matrix, if
 - (1) m < n
- (2) m > n
- (3) m = n
- (4) None of these
- **21.** If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7\hat{A}$ is equal to
 - **(1)** A
- (2) I A
- (3) I
- (4) 3A
- **22.** Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

- (1) $x = \frac{-1}{3}$, y = 7 (2) Not possible to find
- (3) y = 7, $x = \frac{-2}{3}$ (4) $x = \frac{-1}{3}$, $y = \frac{-2}{3}$
- **23.** Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. The restriction on n, k and p so that PY + WY will be defined are:
 - (1) k = 3, p = n
- (2) k is arbitrary, p = 2
- (3) p is arbitrary, k = 3 (4) k = 2, p = 3
- **24.** If A and B are symmetric matrices of same order, AB - BA is a:
 - (1) Skew-symmetric matrix
 - (2) Symmetric matrix
 - (3) Zero matrix
 - (4) Identity matrix
- **25.** The function $f(x) = 2x^3 3x^2 12x + 4$, has
 - (1) two points of local maximum
 - (2) two points of local minimum
 - (3) one maxima and one minima
 - (4) no maxima or minima
- **26.** The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 26x + 15$. The marginal revenue, when x = 15 is:
 - **(1)** ₹ 100
- **(2)** ₹ 115
- (3) ₹ 123
- **(4)** None
- **27.** The maximum profit that a company can make, if the profit function is given by $p(x) = 41 - 72x - 18x^2$ is:
 - **(1)** 111
- **(2)** 112
- (3) 113
- **(4)** 114
- **28.** If $y = x^3 \log x$, then $\frac{d^4 y}{dx^4}$ is:
 - **(1)** 6*x*

- **(4)** log 6
- **29.** If *x* is real, the minimum value of $x^2 8x + 17$ is
 - **(1)** –1
- **(2)** 0
- **(3)** 1
- **(4)** 2

- **30.** A candidate claims 70% of the people in her constituency would vote for her. If 1,20,000 valid votes are polled, then the number of votes she expects from her constituency is
 - **(1)** 100000
- (3) 56000
- (4) 36000
- **31.** Given that $x = at^2$ and y = 2at then $\frac{d^2y}{dx^2}$ is
 - $\overline{2at^3}$
- $2at^2$

- (3) $\frac{1}{t^2}$ (4) $\frac{-2a}{t}$ 32. The expectation of a random variable X (continuous or discrete) is given by......
 - (1) $\sum X f(x)$, $\int X f(X)$
- (2) $\sum X^2 f(X), \int X^2 f(X)$
- (3) $\sum f(X)$, $\int f(X)$
- (4) $\sum X f(X^2), \int X f(X^2)$
- **33.** Mean of a constant 'a' is
 - **(1)** 0
- (3) a/2
- **(4)** 1
- **34.** Find the expectation of a random variable *X*.

	_			
X	0	1	2	3
f(X)	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{6}$
(1) 0.5	•	(2) 1	.5	

- (4) 3.5
- (3) 2.5
- **35.** Skewness of Normal distribution is (2) Positive
 - (1) Negative **(3)** 0
- (4) Undefined
- **36.** Which of the following values is used as a summary measure for a sample, such as a sample mean?
 - (1) Population Parameter
 - (2) Sample Parameter
 - (3) Sample Statistic
 - (4) Population mean
- **37.** A simple random sample consist of four observation 1, 3, 5, 7. What is the point estimate of population standard deviation?
 - **(1)** 2.3
- **(2)** 2.52
- **(3)** 0.36
- **(4)** 0.4
- **38.** A price index which is based on the prices of the items in the composite, weighted by their relative index is called:
 - (1) price relatives
 - (2) Consumer price index
 - (3) Weighted aggregative price index
 - (4) Simple aggregative index
- **39.** Which of the following is an example of line series problem?
 - (i) Estimating numbers of hotel rooms booking in next 6 months.
 - Estimating the total sales in next 3 years of an insurance company.
 - (iii) Estimating the number of calls for the next one week.
 - (1) Only (iii)
- (2) (i) and (ii)
- (3) (i), (ii) and (iii)
- (4) (ii) and (iii)

- **40.** In Paasche's price index number weight is considered as
 - (1) Quantity in base year
 - (2) Quantity in current year
 - (3) Prices in base year
 - (4) Prices in current year
- **41.** Moving average method is used for measurement of trend when:
 - (1) Trend is linear
 - (2) Trend is non-linear
 - (3) Trend is curvilinear
 - (4) None of these
- **42.** The present value of a sequence of payment of ₹ 1000 made at the end of every 6 months and continuing forever, if money is worth 8% per annum compounded semi-annually is
 - **(1)** 1000
- **(2)** 2500
- (3) 25,000
- **(4)** 15,000
- **43.** Assume that Shyam holds a perpetual bond that generates an annual payment of ₹ 500 each year. He believes that the borrower is creditworthy and that an 8% interest rate will be suitable for this bond. The present value of this perpetuity is
 - **(1)** ₹ 6520
- **(2)** ₹ 6250
- (3) ₹ 5620
- (4) ₹ 2650
- **44.** Feasible region in the set of points which satisfy
 - (1) The objective functions
 - (2) Some the given constraints
 - (3) All of the given constraints
 - (4) None of these
- **45.** $Z = 20x_1 + 20x_2$, subject to $x_1 \ge 0$, $x_2 \ge 0$, $x_1 + 2x_2 \ge 8$, $3x_1 + 2x_2 \ge 15$, $5x_1 + 2x_2 \ge 20$. The minimum value of Z occurs at
 - **(1)** (8, 0)
- (3) $\left(\frac{7}{2}, \frac{9}{4}\right)$

Read the following text and answer the following questions on the basis of the same :

Rohan has completed his MBA and now he wants to start a new business. So, he approaches to many banks. One bank is agreed to give loan to Rohan. So, Rohan has borrowed ₹ 5 lakhs from a bank on the interest rate of 12 per cent for 10 years.

- **46.** EMI stands for:
 - (1) Equated Monthly Installments
 - (2) Emerging Monthly Installments
 - (3) Easy Monthly Installments
 - (4) None of the above
- **47.** To calculate monthly installment, we use the following formula:
 - (1) Installment Amount = $\frac{(1+i)^n}{(1+i)^n} \times (P \times i)$
 - (2) Installment Amount = $\frac{(1+i)^n}{(1+i)^n 1} \times (P \times i)$
 - (3) Installment Amount = $\frac{(1+i)^n}{(1+i)^{n-1}} \times (P \times i)$
 - (4) None of these
- **48.** Calculate monthly installment using $(1.01)^{120}$ = 3.300
 - **(1)** ₹7100
- **(2)** ₹7174
- (3) ₹7147
- (4) ₹7200
- **49.** Find the amount of total payment made by Rohan.
 - (1) ₹8,60,88
- **(2)** ₹ 8,80,880
- (3) ₹8,60,000
- **(4)** ₹ 8,60,880
- **50.** Find the amount of interest paid by Rohan.
 - (1) ₹3,60,88
- **(2)** ₹ 3,60,880
- (3) ₹3,60,00
- (4) ₹3,600,88

Section - A

1. Option (3) is correct.

Explanation: Given that,

$$f(x) = \begin{cases} mx + 1 & \text{if } x \le \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$$

is continuous function at $x = \frac{\pi}{2}$, then

$$LHL = RHL$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}} f(x) = \lim_{x \to \frac{\pi}{2}} f(x)$$

$$\Rightarrow \lim_{h \to 0} f\left(\frac{\pi}{2} - h\right) = \lim_{h \to 0} f\left(\frac{\pi}{2} + h\right)$$

$$\Rightarrow \lim_{h \to 0} m\left(\frac{\pi}{2} - h\right) + 1 = \lim_{h \to 0} \sin\left(\frac{\pi}{2} + h\right) + n$$

$$\Rightarrow \lim_{h \to 0} m\left(\frac{\pi}{2} - h\right) + 1 = \lim_{h \to 0} \cos h + n$$

$$\Rightarrow m\left(\frac{\pi}{2}\right) + 1 = 1 + n$$

$$\Rightarrow n = \frac{m\pi}{2}$$

2. Option (3) is correct.

Explanation:

$$\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$$

On expanding along R₁

$$2(x-9x) - 3(x-4x) + 2(9x-4x) + 3 = 0$$

$$2(-8x) - 3(-3x) + 2(5x) + 3 = 0$$

$$-16x + 9x + 10x + 3 = 0$$

$$3x + 3 = 0$$

$$3x = -3$$

$$x = -\frac{3}{3}$$

$$x = -1$$

3. Option (1) is correct.

Explanation:

Let
$$I = \int_{0}^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \qquad \dots(i)$$

$$\int_{0}^{a} f(x) dx = \int_{0}^{a} f(a - x) dx$$

$$\therefore I = \int_{0}^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right) dx}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}$$

$$I = \int \frac{\cos x}{\cos x + \sin x} dx \qquad \dots(ii)$$
Adding equations (i) + (ii),
$$\Rightarrow 2I = \int_{0}^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$= \int_{0}^{\pi/2} 1 dx$$

$$= \left[x\right]_{0}^{\pi/2}$$

$$= \frac{\pi}{2}$$

$$\therefore I = \int_{0}^{\pi/2} \frac{\sin x}{\sin x} dx \qquad \dots(ii)$$

Hence R is true.

From (ii), A is also true.

R is the correct explanation for A.

4. Option (1) is correct.

Explanation:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c.$$

This is a standard integral and hence true. So R is true.

$$\int_{0}^{x^{2}} \frac{dt}{t^{2} + 4} = \left[\frac{1}{2} \tan^{-1} \left(\frac{t}{2} \right) \right]_{0}^{x^{2}}$$

$$= \frac{1}{2} \tan^{-1} \left(\frac{x^2}{2}\right)$$

$$\frac{d}{dx} \left[\int_0^{x^2} \frac{dt}{t^2 + 4} \right] = \frac{d}{dx} \left[\frac{1}{2} \tan^{-1} \left(\frac{x^2}{2}\right) \right]$$

$$= \frac{1}{2} \times \frac{1}{1 + \frac{x^4}{4}} \times \frac{2x}{2}$$

$$= \frac{x}{2} \times \frac{4}{4 + x^4}$$

$$= \frac{2x}{4 + x^4}$$

Hence A is true and R is the correct explanation for A.

5. Option (3) is correct.

Explanation: Consider that *aRb*, if *a* is congruent to b, $\forall a, b \in T$.

Then, $aRa \Rightarrow a \cong a$,

Which is true for all $a \in T$

So, *R* is reflexive, ...(i)

Let $aRb \Rightarrow a \cong b$

 $b \cong a$ \Rightarrow

bRa

So, R is symmetric. ...(ii)

Let aRb and bRc

 \Rightarrow $b \cong b$ and $b \cong a$

 \Rightarrow $a \cong c \quad aRc$

So, *R* is transitive ...(iii)

Hence, *R* is equivalence relation.

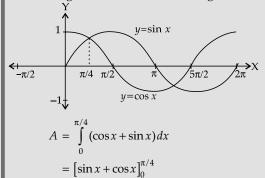
6. Option (3) is correct.

Explanation: We have $y = \cos x$ and $y = \sin x$, where $0 \le x \le \frac{\pi}{2}$.

We get $\cos x = \sin x$

$$x = \frac{\pi}{4}$$

From the figure, area of the shaded region,

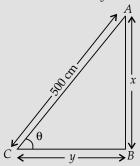


$$= \left[\sin \frac{\pi}{4} + \cos \frac{\pi}{4} - \sin 0 - \cos 0 \right]$$
$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$$
$$= (\sqrt{2} - 1) \text{ sq. units}$$

7. Option (2) is correct.

Explanation: Let the angle between floor and the ladder be θ .

Let AB = x cm and BC = y cm



$$\therefore \qquad \sin \theta = \frac{x}{500} \text{ and } \cos \theta = \frac{y}{500}$$

$$\Rightarrow \qquad x = 500 \sin \theta \text{ and } y = 500 \cos \theta$$

Also,
$$\frac{dx}{dt} = 10 \text{ cm/s}$$

$$\Rightarrow 500.\cos\theta. \frac{d\theta}{dt} = 10 \text{ cm/s}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{10}{500\cos\theta} = \frac{1}{50\cos\theta}$$
For $y = 2 \text{ m} = 200 \text{ cm}$,

$$y = 2 \text{ m} = 200 \text{ cm},$$

$$\frac{d\theta}{dt} = \frac{1}{50 \cdot \frac{y}{500}}$$

$$= \frac{10}{y}$$

$$= \frac{10}{200}$$

$$= \frac{1}{20} \text{ rad/s}$$

8. Option (1) is correct.

Explanation: $P(A) \neq 0$

and

$$P(B|A) = 1$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$1 = \frac{P(B \cap A)}{P(A)}$$

$$P(A) = P(B \cap A)$$

$$A \subset B$$

Explanation:

$$= \sin^{-1} \left[\cos \left(\frac{3\pi}{5} \right) \right]$$

$$= \sin^{-1} \left[\cos \left(\frac{\pi}{2} + \frac{\pi}{10} \right) \right]$$

$$= \sin^{-1} \left(-\sin \frac{\pi}{10} \right) \qquad \left[\because \cos \left(\frac{\pi}{2} + x \right) = -\sin x \right]$$

$$= -\sin^{-1} \left(\sin \frac{\pi}{10} \right) \qquad \left[\because \sin^{-1} (-x) = -\sin^{-1} x \right]$$

$$= -\frac{\pi}{10} \qquad \left[\because \sin^{-1} (\sin x) = x, x \in \left(\frac{-\pi}{2}, \frac{\pi}{2} \right) \right]$$

10. Option (3) is correct.

Explanation:

$$P(A \mid B) > P(A)$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} > P(A)$$

$$\Rightarrow P(A \cap B) > P(A).P(B)$$

$$\Rightarrow \frac{P(A \cap B)}{P(A)} > P(B)$$

$$\Rightarrow P(B \mid A) > P(B)$$

11. Option (3) is correct.

Explanation: We have, the equation of the curve is $3x^2 - y^2 = 8$ (i)

Also, the given equation of the line is x + 3y = 8

$$\Rightarrow \qquad 3y = 8 - x$$

$$\Rightarrow \qquad y = -\frac{x}{3} + \frac{8}{3}$$

Thus, slope of the line is $-\frac{1}{3}$ which should be equal to slope of the equation of normal to the curve.

On differentiating equation (i) with respect to x, we get

$$6x - 2y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{6x}{2y} = \frac{3x}{y} = \text{Slope of the curve}$$

Now, slope of normal to the curve

$$= -\frac{1}{\left(\frac{dy}{dx}\right)}$$
$$= -\frac{1}{\left(\frac{3x}{y}\right)}$$
$$= -\frac{y}{3x}$$

 $3x^2 - x^2 = 8$

On substituting the value of the given equation of the curve, we get

$$\Rightarrow 2x^2 = 8$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$
For
$$x = 2$$

$$3(2)^2 - y^2 = 8$$

$$\Rightarrow y^2 = 4$$

So, the points at which normal is parallel to the given line are $(\pm 2, \pm 2)$.

Hence, the equation of normal at $(\pm 2, \pm 2)$ is

$$\Rightarrow \qquad y - (\pm 2) = -\frac{1}{3}[x - (\pm 2)]$$

$$\Rightarrow \qquad 3[y - (\pm 2)] = -[x - (\pm 2)]$$

$$\therefore \qquad x + 3y \pm 8 = 0$$

12. Option (2) is correct.

Explanation: $aRb \Rightarrow a$ is brother of b.

This does not mean b is also a brother of a as b can be a sister of a.

Hence, *R* is not symmetric.

 $aRb \Rightarrow a$ is brother of b

and $bRc \Rightarrow b$ is a brother of c.

So, a is brother of c.

Hence, *R* is transitive.

13. Option (2) is correct.

Explanation: Corner points Corresponding value of Z = 4x + 3y (0, 0) 0 (0, 40) 120 (20, 40) 200 (60, 20) 300 \leftarrow Maximum (60, 0) 240

Hence, maximum value of Z = 300 < 325 So, the quantity in column *B* is greater.

Explanation: Let,
$$I = \int_{-\pi/2}^{\pi/2} (x^3 + x \cos x + \tan^5 x + 1) dx$$
$$= \int_{-\pi/2}^{\pi/2} x^3 dx + \int_{-\pi/2}^{\pi/2} x \cos x + \int_{-\pi/2}^{\pi/2} \tan^5 x dx + \int_{-\pi/2}^{\pi/2} 1 \cdot dx$$

It is known that if f(x) is an even function, then $\int_{a}^{a} f(x) dx = 2 \int_{a}^{a} f(x) dx$ and if f(x) is an odd function, then

$$\int_{-a}^{a} f(x) dx = 0$$

$$\therefore I = 0 + 0 + 0 + 2 \int_{0}^{\pi/2} 1 \cdot dx$$

$$= 2 \left[x \right]_{0}^{\pi/2} = \frac{2\pi}{2} = \pi$$

15. Option (4) is correct.

Explanation:

The foot of perpendicular from point $P(\alpha, \beta, \gamma)$ on y-axis is $Q(0,\beta,0)$.

:. Required distance,

$$PQ = \sqrt{(\alpha - 0)^2 + (\beta - \beta)^2 + (\gamma - 0)^2} = \sqrt{\alpha^2 + \gamma^2}$$

Section - B1

16. Option (4) is correct.

Explanation: Let,

$$\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right] = \sin^{-1}\left[\cos\left(6\pi + \frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$$

$$[\because \cos(2n\pi + \theta) = \cos\theta]$$

$$= \sin^{-1}\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{10}\right)\right]$$

$$= \sin^{-1}\left(-\sin\frac{\pi}{10}\right)$$

$$\left[\because \cos\left(\frac{\pi}{2} + x\right)\right] = -\sin x$$

$$= -\sin^{-1}\left(\sin\frac{\pi}{10}\right)$$

$$[\because \sin^{-1}(-x) = -\sin^{-1}x]$$

$$= -\frac{\pi}{10}$$

$$[\because \sin^{-1}(\sin x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

17. Option (4) is correct.

Explanation:

Given,
$$y = \log_e \left(\frac{x^2}{e^2}\right)$$

$$\Rightarrow \qquad y = 2\log_e x - \log_e e^2$$

$$\Rightarrow \qquad y = 2\log_e x - 2$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{2}{x}$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \frac{-2}{x^2}$$

18. Option (4) is correct.

Explanation: Given that, $ay + x^2 = 7$ and $x^3 = y$ On differentiating both equations with respect

$$a \cdot \frac{dy}{dx} + 2x = 0 \qquad \text{and} \qquad 3x^2 = \frac{dy}{dx}$$

$$\Rightarrow \qquad \frac{dy}{dx} = -\frac{2x}{a} \qquad \text{and} \qquad \frac{dy}{dx} = 3x^2$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = \frac{-2}{a} = m_1$$

and
$$\left(\frac{dy}{dx}\right)_{(1,1)} = 3.1 = 3 = m_2$$

Since, the curve cuts orthogonally at (1, 1).

$$\begin{array}{ccc} \therefore & m_1 m_2 = -1 \\ \Rightarrow & \left(\frac{-2}{a}\right) . 3 = & -1 \\ \therefore & a = & 6 \end{array}$$

19. Option (4) is correct.

Explanation: $A = \begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 50 & 40 \\ 2 & 3 \end{bmatrix}$ $AB = \begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix} \begin{bmatrix} 50 & 40 \\ 2 & 3 \end{bmatrix}$ $= \begin{bmatrix} 10000 + 100 & 8000 + 150 \\ 500 + 4 & 400 + 6 \end{bmatrix}$

$$AB = \begin{bmatrix} 10100 & 8150 \\ 504 & 406 \end{bmatrix}$$
$$|AB| = (10100)(406) - (504)(8150)$$
$$= 4100600 - 4107600$$
$$= -7000$$

Explanation:

$$P(A) + P(B) - P(A \text{ and } B) = P(A)$$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A)$$

$$\Rightarrow P(B) - P(A \cap B) = 0$$

$$\Rightarrow P(A \cap B) = P(B)$$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B)}{P(B)}$$

$$= 1$$

21. Option (4) is correct.

Explanation: We have, the equation of line as $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$

3 4 5

This line is parallel to the vector, $\vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Equation of plane is 2x - 2y + z = 5. Normal to the plane is $\vec{n} = 2\hat{i} - 2\hat{j} + \hat{k}$.

The angle between line and plane is ' θ '. Then,

$$\sin \theta = \frac{\left| \vec{b} \cdot \vec{n} \right|}{\left| \vec{b} \right| \left| \vec{n} \right|}$$

$$= \frac{\left| (3\hat{i} + 4\hat{j} + 5\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k}) \right|}{\sqrt{3^2 + 4^2 + 5^2} \sqrt{4 + 4 + 1}}$$

$$= \frac{\left| 6 - 8 + 5 \right|}{\sqrt{50} \sqrt{9}}$$

$$= \frac{3}{15\sqrt{2}}$$

$$= \frac{1}{5\sqrt{2}}$$

$$\sin \theta = \frac{\sqrt{2}}{10}$$

22. Option (4) is correct.

Explanation:

Explanation:

$$x^{2} = x + 2$$

$$x^{2} - x - 2 = 0$$

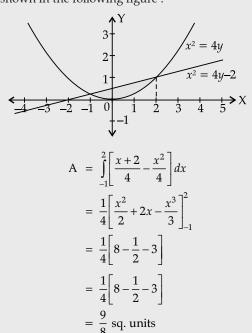
$$(x - 2)(x + 1) = 0$$

$$x = -1, 2$$
For $x = -1$, $y = \frac{1}{4}$ and for $x = 2$, $y = 1$

Points of intersection are $(-1, \frac{1}{4})$ and (2, 1).

Graphs of parabola $x^2 = 4y$ and x = 4y - 2 are

shown in the following figure:



23. Option (4) is correct.

Explanation: We have $\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix} = \begin{vmatrix} a+c & b+c+a & a \\ b+c & c+a+b & b \\ c+b & a+b+c & c \end{vmatrix}$ $[\because C_1 \to C_1 + C_2 \text{ and } C_2 \to C_2 + C_3]$ $= (a+b+c)\begin{vmatrix} a+c & 1 & a \\ b+c & 1 & b \\ c+b & 1 & c \end{vmatrix}$ $[Taking (a+b+c) \text{ common from } C_2]$ $[\because R_2 \to R_2 - R_3 \text{ and } R_1 \to R_1 - R_3]$ $= (a+b+c)\begin{vmatrix} a-b & 0 & a-c \\ 0 & 0 & b-c \\ c+b & 1 & c \end{vmatrix}$

[Expanding along R_2] = (a+b+c)[(b-c)(a-b)]= (a+b+c)(b-c)(a-b)

24. Option (3) is correct.

Explanation: The repeated selections of defective bulbs from a box are Bernoulli trials. Let X denotes the number of defective bulbs out of a sample of 5 bulbs.

Probability of getting a defective bulb,

$$p = \frac{10}{100}$$

$$= \frac{1}{10}$$

$$\therefore \qquad q = 1 - p$$

$$= 1 - \frac{1}{10}$$

$$= \frac{9}{10}$$
Clearly When a bin which is

 $= \frac{9}{10}$ Clearly, *X* has a binomial distribution with n = 5and $p = \frac{1}{10}$.

$$P(X = x) = {}^{n}C_{x}q^{n-x}p^{x}$$
$$= {}^{5}C_{x}\left(\frac{9}{10}\right)^{5-x}\left(\frac{1}{10}\right)^{x}$$

P (none of the bulbs is defective) = P(X=0)

$$= {}^{5}C_{0} \cdot \left(\frac{9}{10}\right)^{5}$$
$$= 1 \cdot \left(\frac{9}{10}\right)^{5}$$
$$= \left(\frac{9}{10}\right)^{5}$$

25. Option (4) is correct.

Explanation: The degree of above differential equation is not defined because when we expand $\sin\left(\frac{dy}{dx}\right)$ we get an infinite series in the increasing powers of $\frac{dy}{dx}$. Therefore its degree is not defined.

26. Option (2) is correct.

Explanation: Given that,

$$f(x) = |2x - 1| \sin x$$

The function $\sin x$ is differentiable.

The function |2x-1| is differentiable, except

$$2x - 1 = 0$$
$$x = \frac{1}{2}$$

Thus, the given function is differentiable $R - \left\{ \frac{1}{2} \right\}$

27. Option (4) is correct.

Explanation: Given that the equation of curve is $y(1+x^2) = 2-x$

On differentiating with respect to x, we get

$$y.(0+2x)+(1+x^2).\frac{dy}{dx}=0-1$$

$$\Rightarrow 2xy + (1+x^2)\frac{dy}{dx} = -1$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1-2xy}{1+x^2} \qquad ...(ii)$$

Since, the given curve passes through *x*-axis,

i.e.,
$$y = 0$$

$$0(1+x^2) = 2-x$$
[By using Eq. (i)] $\Rightarrow x = 2$

So the curve passes through the point (2, 0).

$$\left(\frac{dy}{dx}\right)_{(2, 0)} = \frac{-1 - 2 \times 0}{1 + 2^2} = -\frac{1}{5}$$
= Slope of the curve

- ∴ Slope of tangent to the curve = $-\frac{1}{2}$
- .. Equation of tangent to the curve passsing through (2,0) is

$$y - 0 = -\frac{1}{5}(x - 2)$$

$$\Rightarrow \qquad y + \frac{x}{5} = +\frac{2}{5}$$

$$\Rightarrow \qquad 5y + x = 2$$

28. Option (2) is correct.

Explanation: R on the set {1, 2, 3} is defined by

That shows neither reflexive nor symmetric but transitive.

29. Option (1) is correct.

Explanation:

The distance of the plane $\vec{r} \cdot \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 1$ from the origin is 1.

[Since $\vec{r} \cdot \hat{n} = d$ is the form of above equation, where d represents the distance of plane from the origin, *i.e.*, d = 1]

30. Option (2) is correct.

Explanation:

Explanation:

$$x = \tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$$
Let $\cos^{-1}\frac{\sqrt{5}}{3} = \theta$

$$\cos\theta = \frac{\sqrt{5}}{3}$$

$$\Rightarrow x = \tan\frac{1}{2}\theta$$

$$\sin\frac{\theta}{2}$$

$$\Rightarrow \qquad x = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$$

$$\therefore \qquad \sin\frac{\theta}{2} = \frac{\sqrt{1 - \frac{\sqrt{5}}{3}}}{\sqrt{2}}$$

$$\Rightarrow \qquad \cos\frac{\theta}{2} = \frac{\sqrt{1 + \frac{\sqrt{5}}{3}}}{\sqrt{2}}$$

$$x = \frac{\sqrt{1 - \frac{\sqrt{5}}{3}}}{\sqrt{1 + \frac{\sqrt{5}}{3}}}$$

$$= \frac{\sqrt{3 - \sqrt{5}}}{\sqrt{3 + \sqrt{5}}}$$

$$= \frac{\sqrt{3 - \sqrt{5}}}{\sqrt{3 + \sqrt{5}}} \times \frac{\sqrt{3 - \sqrt{5}}}{\sqrt{3 - \sqrt{5}}}$$

$$= \frac{3 - \sqrt{5}}{\sqrt{(3)^2 - (\sqrt{5})^2}}$$

$$= \frac{3 - \sqrt{5}}{\sqrt{9 - 5}}$$

$$= \frac{3 - \sqrt{5}}{2}$$

Explanation:

Let
$$I = \frac{\cos 2x}{(\cos x + \sin x)^2}$$

$$I = \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} dx$$

$$= \int \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2} dx$$

$$= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$$
Let
$$\cos x + \sin x = t$$

$$\Rightarrow (\cos x - \sin x) dx = dt$$

$$\Rightarrow I = \int \frac{dt}{t}$$

$$= \log|t| + C$$

$$= \log|\cos x + \sin x| + C$$

32. Option (4) is correct.

Explanation: Given that, $A = \{1, 2, 3\}$

Now, number of equivalence relations are as follows:

$$R_1 = \{(1, 1), (2, 2), (3, 3)\}$$

$$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$$

$$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$$

$$\begin{array}{ll} R_4 = \{(1,1),(2,2),(3,3),(2,3),(3,2)\} \\ R_5 = \{(1,2,3) \Leftrightarrow A \times A = A^2\} \end{array}$$

 \therefore Maximum number of equivalence relations on the set $A = \{1, 2, 3\} = 5$

33. Option (2) is correct.

Explanation:

Let *X* be the random variable representing a number on the die.

The total number of observations is 6. Therefore,

$$P(X = 1) = \frac{3}{6}$$

$$= \frac{1}{2}$$

$$P(X = 2) = \frac{2}{6}$$

$$P(X = 1) = \frac{3}{6}$$

$$= \frac{1}{2}$$

$$P(X = 2) = \frac{2}{6}$$

Therefore, the probability distribution is as follows.

X	1	2	5	
P(X)	1/2	1/3	1/6	
	Mean = E(X)			
$=\sum p_i x_i$				
	$=\frac{1}{2}\times$	$1 + \frac{1}{3} \times 2 + \frac{1}{6}$	×5	
	$=\frac{1}{2}+$	$\frac{2}{3} + \frac{5}{6}$		
	=	$\frac{4+5}{6}$		
$=\frac{12}{6}$				
	= 2			

34. Option (3) is correct.

Explanation:

Given that, $y = e^{-x} (A\cos x + B\sin x)$

On differentiating both sides w.r.t., x we get

$$\frac{dy}{dx} = -e^{-x}(A\cos x + B\sin x)$$
$$+e^{-x}(-A\sin x + B\cos x)$$
$$\frac{dy}{dx} = -y + e^{-x}(-A\sin x + B\cos x)$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{-dy}{dx}$$

$$+e^{-x}(-A\cos x - B\sin x)$$

$$-e^{-x}(-A\sin x + B\cos x)$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = -\frac{dy}{dx} - y\left[\frac{dy}{dx} + y\right]$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = -\frac{dy}{dx} - y - \frac{dy}{dx} - y$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = -2\frac{dy}{dx} - 2y$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

Explanation: When two dices are rolled, the number of outcomes is 36. The only even prime number is 2.

Let E be the event of getting an even prime number on each die.

$$E = \{(2, 2)\}$$

$$\Rightarrow P(E) = \frac{1}{36}$$

36. Option (1) is correct.

Explanation:

$$\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}}{2}$$
$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{\vec{b}}{2}$$

Compare

Compare
$$\Rightarrow \qquad \vec{a} \cdot \vec{b} = 0$$
&\tag{ab} \cos \alpha = 0
\tag{cos } \alpha = 0
\tag{cos } \alpha = 0
\tag{cos } \alpha = \cos \left(\frac{\pi}{2} \right)
\tag{ac} \tag{cos } \beta = \frac{1}{2}
\tag{cos } \beta = \frac{1}{2}
\tag{cos } \beta = \cos \left(\frac{\pi}{3} \right)
\tag{\beta} \tag{\beta} = \frac{60^{\circ}}{3}

 $\vec{a} \& \vec{b}$ are unit vectors

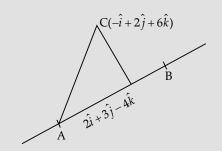
 \Rightarrow

$$\begin{vmatrix} \alpha - \beta \end{vmatrix} = \begin{vmatrix} 90^{\circ} - 60^{\circ} \end{vmatrix}$$
$$= 30^{\circ}$$

37. Option (1) is correct.

Explanation:

$$\vec{r} = (2\hat{i} + 3\hat{j} - 4\hat{k}) + \lambda(6\hat{i} + 3\hat{j} - 4\hat{k})$$



AB: Projection of AC on AB

Projection of AC on AB
$$\overline{AC} = (-\hat{i} + 2\hat{j} + 6\hat{k}) - (2\hat{i} + 3\hat{j} - 4\hat{k})$$

$$= -3\hat{i} - \hat{j} + 10\hat{k}$$

$$\overline{AB} = \frac{|\overline{AB} \cdot \overline{AC}|}{|\overline{AB}|}$$

$$= \frac{|-18 - 3 - 40|}{\sqrt{61}}$$

$$= \sqrt{61}$$

$$|\overline{AC}| = 110$$

$$BC = \sqrt{AC^2 - AB^2}$$

38. Option (2) is correct.

Explanation: Given that,

$$y = \log\left(\frac{1 - x^2}{1 + x^2}\right)$$

$$\Rightarrow y = \log\left(1 - x^2\right) - \log\left(1 + x^2\right).$$

Differentiate with respect to x, we have

$$\frac{dy}{dx} = \frac{d}{dx} \Big[\log(1 - x^2) \Big] - \frac{d}{dx} \Big[\log(1 + x^2) \Big]$$

$$= \frac{-2x}{1 - x^2} - \frac{2x}{1 + x^2}$$

$$= -2x \left(\frac{2}{(1 - x^2)(1 + x^2)} \right)$$

$$= -\frac{4x}{1 - x^4}$$

39. Option (1) is correct.

Explanation:
$$\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}$$

$$\vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$$
a vector \perp to both $\vec{a} + \vec{b} & \vec{a} - \vec{b}$ is
$$\lambda\{(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})\}$$

$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$$

$$= (16\hat{i} - 16\hat{j} - 8\hat{k})\lambda \quad ...(i)$$

$$|\vec{c}| = 12 \qquad \text{(given)}$$

$$|\lambda(16\hat{i} - 16\hat{j} - 8\hat{k})| = 12$$

$$\lambda\sqrt{16^2 + 16^2 + 8^2} = 12$$

$$\lambda = \frac{1}{2}$$
From eqn. (i),
$$\vec{c} = \frac{1}{2}(16\hat{i} - 16\hat{j} - 8\hat{k})$$

$$= 4(2\hat{i} - 2\hat{j} - \hat{k})$$

Explanation:

For coplanar vectors

$$\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$$

$$\Rightarrow \qquad \mu(\mu^2 - 1) - 1(\mu - 1) + 1(1 - \mu) = 0$$
$$\mu^3 - 3\mu + 2 = 0$$

$$\mu^{2}(\mu-1) + \mu(\mu-1) - 2(\mu-1) = 0$$

 $\mu(\mu-1)-2(\mu-1)=0$ (By remainder theorem)

$$(\mu^2 + \mu - 2)(\mu - 1) = 0$$

 $\mu = 1, 1, -2$

Sum of distinct value of μ is

$$-2+1 = -1$$

41. Option (2) is correct.

Explanation:

Corner points	Corresponding value of $Z = 3x - 4y$
(0, 0)	0

(5, 0)	15 ← Maximum
(6, 5)	-2
(6, 8)	-14
(4, 10)	-28
(0, 8)	–32 ← Minimum

Hence, the minimum of Z occurs at (0, 8) and its minimum value is (-32).

42. Option (1) is correct.

Explanation:

$$\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b},$$

$$\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$$

$$\vec{\alpha} = \mu \vec{\beta}$$

Because $\vec{\alpha} \& \vec{\beta}$ are collinear

$$(\lambda - 2)\vec{a} + \vec{b} = \mu[(4\lambda - 2)\vec{\alpha} + 3\vec{b}]$$

$$\vec{a} \rightarrow \lambda - 2 = (4\lambda - 2)\mu$$

$$(\lambda - 2) = (4\lambda - 2)^{\frac{1}{2}}$$

$$\Rightarrow \qquad (\lambda - 2) = (4\lambda - 2)\frac{1}{3}$$

$$\Rightarrow \qquad \lambda = -4$$

$$\Rightarrow \qquad \qquad \lambda = -4$$

$$\vec{b} \rightarrow \qquad \qquad 1 = 3\mu$$

$$\Rightarrow \qquad \qquad \mu = \frac{1}{3}$$

43. Option (2) is correct.

Explanation: Given that,

$$f(x) = |\sin x|$$

The functions |x| and $\sin x$ are continuous function for all real value of x.

Thus, the function $f(x) = |\sin x|$ is continuous function everywhere.

Now, |x| is non-differentiable function at x = 0. Since $f(x) = |\sin x|$ is non-differentiable function at $\sin x = 0$

Thus, f is everywhere continuous but not differentiable at $x = n\pi$, $n \in Z$.

44. Option (3) is correct.

Explanation: Given that,

$$\cos x \frac{dy}{dx} + y \sin x = 1$$

$$\Rightarrow \frac{dy}{dx} + y \tan x = \sec x$$

Here, $P = \tan x$ and $Q = \sec x$

$$IF = e^{\int Pdx}$$

$$= e^{\int \tan x dx}$$

$$= e^{\log \sec x}$$

$$IF = \sec x$$

45. Option (4) is correct.

Explanation:

Since, direction cosines of a line are k, k and k. $\therefore l = k$, m = k and n = k

We know that,
$$l^{2} + m^{2} + n^{2} = 1$$

$$\Rightarrow \qquad k^{2} + k^{2} + k^{2} = 1$$

$$\Rightarrow \qquad k^{2} = \frac{1}{3}$$

$$\therefore \qquad k = \pm \frac{1}{\sqrt{3}}$$

Explanation: We know that, area of a triangle with vertices (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is given by

given by
$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore \qquad \Delta = \frac{1}{2} \begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}$$
[Expanding along R_1]
$$9 = \frac{1}{2} [-3(-k) - 0 + 1(3k)]$$

$$\Rightarrow \qquad 18 = 3k + 3k = 6k$$

$$\therefore \qquad k = \frac{18}{6} = 3$$

47. Option (1) is correct.

Explanation: Maximum of Z occurs at (5, 0).

48. Option (1) is correct.

Explanation: According to question,

$$(x-8)(y+10) = xy$$

$$\Rightarrow xy + 10x - 8y - 80 = xy$$

$$\Rightarrow 5x - 4y = 40 \qquad ...(i)$$
and
$$(x+16)(y-10) = xy$$

$$\Rightarrow xy - 10x + 16y - 160 = xy$$

$$\Rightarrow 5x - 8y = -80 \qquad ...(ii)$$

49. Option (3) is correct.

Explanation: Given equations are;

$$5x - 4y = 40$$

$$5x - 8y = -80$$

$$\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

50. Option (4) is correct.

Option (4) is correct.

Explanation: Since,
$$\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$
Let
$$kA = \begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}, x = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$B = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B \qquad ...(iii)$$

$$|A| = 5(-8) - (-4) \times 5$$

$$= -40 + 20$$

$$= -20$$

$$adj(A) = \begin{bmatrix} -8 & -5 \\ 4 & 5 \end{bmatrix}^{T}$$

$$= \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{Adj(A)}{|A|}$$

$$= \frac{1}{-20} \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{2}{-20} \begin{bmatrix} -1 \\ -3 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{2}{5} & -1 \\ \frac{1}{4} & -1 \\ 4 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} \frac{2}{5} \times 40 - \frac{1}{5} \times (-80) \\ \frac{1}{4} \times 40 - \frac{1}{4} \times (-80) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{6} + 16 \\ 10 + 20 \end{bmatrix}$$

$$= \begin{bmatrix} 32 \\ 30 \end{bmatrix}$$

$$\therefore \qquad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$$

$$\therefore \qquad x = 32$$

Hence, 32 children were given some money by Seema.

u = 30

Section - B2

(APPLIED MATHEMATICS)

16. Option (3) is correct.

Explanation: If a and b are integers and n > 0, then we write $a \equiv b \pmod{n}$ to mean $n \mid (a - b)$.

17. Option (2) is correct.

Explanation: $(9 + 23) \mod 12$ or, $32 \mod 12$ On dividing 32 by 12, we get remainder = 8 Hence, $(9 + 23) \mod 12 = 8$

18. Option (4) is correct.

Explanation: As, B > A $\Rightarrow A < B$ $\Rightarrow A - B < 0$ A + B > 0 and AB > 0If A = 1, B = 4 then, AB < A + BIf A = 2, B = 4 then, AB > A + BThus, we can't say which one of A + B and AB has higher value.

19. Option (3) is correct.

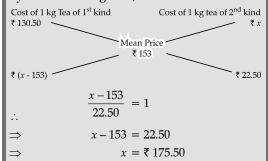
Explanation: Since first and second varieties are mixed in equal proportions.

So, their average price =
$$\mathbf{E}\left(\frac{126+135}{2}\right)$$

= \mathbf{E} 130 50

So, the mixture is formed by mixing two varieties, one at ₹ 130.50 per kg and the other at say ₹ x per kg in the ratio 2 : 2, i.e., 1 : 1. We have to find x.

By the rule of alligation, we have:



20. Option (3) is correct.

Explanation: It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore,

 $A = [a_{if}]_{x=0}$ is a square matrix, if m = n.

21. Option (3) is correct.

Explanation: $(I+A)^3 - 7A = I^3 + A^3 + 3I^2A + 3A^2I - 7A$ $= I + A^3 + 3A + 3A^2 - 7A$ $= I + A^2 \cdot A + 3A + 3A - 7A$ $\left[\because A^2 = A\right]$ $= I + A \cdot A - A$ $= I + A^2 - A$ = I + A - A = I $\therefore (I+A)^3 - 7A = I$

22. Option (2) is correct.

Explanation: It is given that $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Equating the corresponding elements, we get:

$$3x + 7 = 0$$

$$x = -\frac{7}{3}$$

$$5 = y - 2$$

$$\Rightarrow \qquad y = 7$$

$$y + 1 = 8$$

$$\Rightarrow \qquad y = 7$$

$$2 - 3x = 4$$

$$\Rightarrow \qquad x = -\frac{2}{3}$$

We find that on comparing the corresponding elements of the two matrices, we get two different values of x, which is not possible. Hence, it is not possible to find the values of x and y for which the given matrices are equal.

23. Option (1) is correct.

Explanation: Matrices P and Y are of the orders $p \times k$ and $3 \times k$, respectively. Therefore, matrix PY will be defined if k = 3. Consequently, PY will be of the order $p \times k$. Matrices W and Y are of the orders $n \times 3$ and $3 \times k$ respectively. Since the number of columns in W is equal to the number of rows in Y, matrix WY is well-defined and is of the order $n \times k$. Matrices PY and WY can be added only when their orders are the same.

However, PY is of the order $p \times k$ and WY is of the order $n \times k$. Therefore, we must have p = n. Thus, k = 3 and p = n are the restrictions on n, k, and p so that PY + WY will be defined.

Explanation: Given that,
A and B are symmetric matrices.

$$\Rightarrow A = A' \text{ and } B = B'$$
Now, $(AB - BA)' = (AB)' - (BA)'$...(i)

$$\Rightarrow (AB - BA)' = B'A' - A'B'$$
[By reversal law]

$$\Rightarrow (AB - BA)' = BA - AB$$
 [From Eq. (i)]

$$\Rightarrow (AB - BA)' = -(AB - BA)$$

25. Option (3) is correct.

So, f(x) has one maxima and one minima.

26. Option (2) is correct.

Explanation: There
$$R(x) = 3x^{2} + 26x + 15$$

$$\frac{dR}{dx} = 6x + 26$$

$$\Rightarrow \left(\frac{dR}{dx}\right)_{\text{at } x = 15} = 6(15) + 26$$

$$= ₹ 116$$

27. Option (3) is correct. Explanation:

$$p(x) = 41 - 72x - 18x^{2}$$

$$p'(x) = -72 - 36x$$

$$\Rightarrow x = -\frac{72}{36} = -2$$

$$p''(x) = -36$$
Also,
$$p''(-2) = -36 < 0$$
By second derivative test, $x = -2$ is the point of local maxima of p .

=41-72(-2)-18(-2)2=41+144-72=113

 \therefore Maximum profit = p(-2)

Hence, the maximum profit that the company can make is 113 units.

28. Option (2) is correct.

Explanation:
If
$$y = x^3 \log x$$

 $\frac{dy}{dx} = \frac{d}{dx} (x^3 \log x)$
 $\therefore \frac{dy}{dx} = x^2 (1 + 3 \log x)$

$$\frac{d^2y}{dx^2} = x^2 \frac{d}{dx} (1 + 3\log x) + (1 + 3\log x) \frac{d}{dx} x^2$$

$$\frac{d^2y}{dx^2} = x (5 + 6\log x)$$

$$\frac{d^3y}{dx^3} = x \frac{d}{dx} (5 + 6\log x) + (5 + 6\log x) \frac{d}{dx} x$$

$$\frac{d^3y}{dx^3} = 11 + 6\log x$$

$$\frac{d^4y}{dx^4} = \frac{6}{x}$$

29. Option (3) is correct.

Explanation: Let,

$$f(x) = x^2 - 8x + 17$$
On differentiating with respect to x , we get
$$f'(x) = 2x - 8$$
So,
$$f'(x) = 0$$

$$\Rightarrow 2x - 8 = 0$$

$$\Rightarrow 2x = 8$$

$$\therefore x = 4$$
Now, Again on differentiating w.r.t. x , we get
$$f''(x) = 2 > 0, \forall x$$
So, $x = 4$ is the point of local minima.
Minimum value of $f(x)$ at $x = 4$

$$f(4) = 4 \times 4 - 8 \times 4 + 17 = 1$$

30. Option (2) is correct.

Explanation: Expected number of votes
$$= np$$

$$= \frac{70}{100} \times 120000$$

$$= 84000$$

31. Option (1) is correct.

Explanation:
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{2a}{2at}$$

$$= \frac{1}{t}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{t^2} \times \frac{dt}{dx}$$

$$= -\frac{1}{2at^3}$$

32. Option (1) is correct.

Explanation: The expectation of a random variable X is given by the summation (integral) of X times the function in its interval. If it is a continuous random variable, then summation is used and if it is discrete random variable, then integral is used.

Explanation: Let f(x) be the pdf of the random variable X.

Now,
$$E(q) = \int a f(x) = a \int f(x) = a.1 = a$$

34. Option (2) is correct.

Explanation:
$$E(X)$$

= $0\left(\frac{1}{6}\right) + 1\left(\frac{2}{6}\right) + 2\left(\frac{2}{6}\right) + 3\left(\frac{1}{6}\right)$
= $0 + \frac{2}{6} + \frac{4}{6} + \frac{3}{6}$
= $\frac{9}{6} = \frac{3}{2} = 1.5$

35. Option (3) is correct.

Explanation: Since the normal curve is symmetric about its mean, its skewness is zero.

36. Option (3) is correct.

37. Option (2) is correct.

Explanation: The point estimation of population standard deviation is sample deviation.

$$S = \sqrt{\frac{(x_i - \overline{x})^2}{n - 1}}$$

$$= \sqrt{\frac{(1 - 4)^2 + (3 - 4)^2 + (5 - 4)^2 + (7 - 4)^2}{4 - 1}}$$

$$\left(\because \overline{x} = \frac{\Sigma x}{n} = \frac{16}{4} = 4\right)$$

$$= \sqrt{\frac{9 + 1 + 1 + 9}{2}} = \sqrt{\frac{20}{3}} = 2.52$$

38. Option (3) is correct.

39. Option (3) is correct.

Explanation: All the component are associated with time.

40. Option (2) is correct.

41. Option (1) is correct.

Explanation: When trend is linear, then only we use moving average method for measurement.

42. Option (3) is correct.

Explanation: The given annuity is a perpetuity. present value of perpetuity =
$$\frac{\text{Cash flow}}{\text{Interest rate}}$$

Here, case flow = $\frac{?}{1000}$

interest rate = $\frac{8/2}{100}$

$$= \frac{4}{100} = 0.04$$
So, present value = $\frac{1000}{0.04}$
= ₹ 25,000

43. Option (2) is correct.

Explanation: PV of perpetuity
$$= \frac{\text{Annual Payment/Cash flow}}{\text{Interest rate/yield}}$$

$$= \frac{500}{\frac{8}{100}} = \frac{500}{0.08}$$

$$= ₹ 6250$$

44. Option (3) is correct.

45. Option (3) is correct.

Explanation:	
Corner Points	$Z = 20x_1 + 20x_2$
(8, 0)	160
$\left(\frac{5}{2},\frac{15}{4}\right)$	125
$\left(\frac{7}{2},\frac{9}{4}\right)$	115 (minimum)
(0, 10)	200

46. Option (1) is correct.

47. Option (2) is correct.

Explanation: Formula to calculate monthly installment is:

Installment Amount =
$$\frac{(1+i)^n}{(1+i)^n - 1} \times (P \times i)$$

48. Option (3) is correct.

Given, $i = \frac{\left(\frac{\text{annual rate}}{12}\right)}{100}$ $= \left[\frac{\left(\frac{12}{12}\right)}{100}\right]$ $= \frac{1}{100}$ = 0.01 $n = 10 \times 12$ = 120

P = ₹ 5,00,000

Installment Amount
$$= \frac{(1+i)^n}{(1+i)^n - 1} \times (P \times i)$$
Installment Amount
$$= \frac{(1+0.01)^{120}}{(1+0.01)^{120} - 1}$$

$$\times (5,00,000 \times 0.01)$$

$$= \frac{3.300}{3.300 - 1} \times 5,000$$

$$= \frac{16,500}{2.300}$$

$$= ₹ 7173.91 \sim ₹ 7174$$
So, EMI that Rohan has to pay is ₹ 7147

Explanation: Total payment made by Rohan to the bank in 10 years = $(EMI \times Total tenure in$ months) = ₹ (7174 × 120) = ₹ 8,60,880

50. Option (1) is correct.

Explanation: The total interest amount payable will be (Total payment – loan amount) = ₹ 3,60,88.