MATHEMATICS (Code – 14)

Time : 3 Hours

Maximum Marks : 150

Note: Attempt FIVE questions in all. All questions carry equal marks. Question No. 1 is compulsory. Answer any two questions from Part-I and any two questions from Part-II. The parts of the same question must be answered together and must not be interposed between answers to other questions.

1. Answer any four of the following. **(4 x 7.5)**

(a) Find the supremum and infimum,if they exit of the following set

(i) $\left\{x: x = 1 + \left(\frac{1}{n}\right), n \in N\right\}$ (ii) $\left\{x: x = 1 - \left(\frac{1}{n}\right), n \in N\right\}$

(b) Let V be the vector space of all polynomials. Consider the subspace W spanned by $t^2 + t + 2, t^2 + 2t + 5, 5t^2 + 3t + 4$ *and* $2t^2 + 2t + 4$.Then find the dimension of W.

(c) If two balls are drown from a bag containing 2 white, 4 red and 5 black balls. What the chance that (i) both the balls are red (ii) one is red and other is black.

(d) If $\theta \in R\{n\pi : n \in Z\}$ and P is a (2x2) matrix with complex entries such that $P^{-1}\begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} P = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}$ then find the matrix P.

(e) If $y = \varphi(x)$ is a particular solution of $y'' + (sinx)y' + 2y = e^x$ and $y = \omega(x)$ is a particular solution of $y'' + (sinx)y' + 2y = cos2x$,then find a particular solution of $y'' + (sinx)y' + 2y = 2sin^2x + e^x$,

(f) Find an interval of unit length which contain the smallest positive root of the equation $f(x) = x^3 - 5x - 1$, hence determine the number of iterations required by the bisection method so that $|error| < 10^{-2}$.

Part I

2. (a) Let V and W be finite dimensional vector space. Let $T: V \to W$ be a linear transformation and $\{u_1, u_2, u_3, \dots, u_n\}$ be a subset of V such that set $\{Tu_1, Tu_2, Tu_3, \dots \dots, Tu_n\}$ is a linearly independent in W. Then show that set $\{u_1, u_2, u_3, \dots, u_n\}$ is linearly independent in V. **(10)**

(b) Evaluate the double integral $\iint e^{x^2}\,dxdy$ over the region R , given by R: $2y \le x \le 2$ and $0 \le y \le 1$. **(10)**

(c) Let $f: R^2 \to R$ be defined by

$$f(x,y) = \frac{x^2 y}{x^4 + y^2}, (x,y) \neq (0,0) \text{ and } f(x,y) = 0, (x,y) = (0,0).$$

Then Find the directional derivative of $f(x,y)$ at (0,0) in the direction f of the vector $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$. **(10)**

3. (a) Find the directrix of the parabola $r = \frac{25}{10+10\cos\theta}$ **(10)**

(b) Solve the $(D^4 + 2D + 1)y = x^2 \cos x, D \equiv \frac{d}{dx}$ **(12)**

(c) Solve $(x^2 - 1)\left(\frac{dy}{dx}\right) + 2xy = 1$ **(8)**

4. (a) A bead of weight W can slide on a smooth circular wire in a vertical plane, the bead is attached by a light thread to the highest point of the wire, and in equilibrium the thread is taut then find the tension of the thread and the reaction of the wire on the bead, if the length of the string is equal to the radius of the wire. **(10)**

(b) For (1/m) of the distance between two stations, a train is uniformly accelerated and for (1/n) of the distance, it is uniformly retarded. It starts from rest at one station and comes to rest at the other. Find the ratio of its greatest velocity to its average velocity. **(10)**

(c) A particle of mass 'm' is oscillating in a straight line about a centre of force 'o' towards which the force is mn^2r when at a distance r. The particle receives a bend in the direction of motion which generates a velocity na, when it is at a distance $a\sqrt{3}/2$ from 'o', a is the amplitude of the oscillation. If this velocity is away from 'o', show that the new amplitude is $a\sqrt{3}$.

(10)

Part II

5. (a) It is required to solve the system SX=T, where $S=\begin{pmatrix} 2 & -1 & 1 \\ 2 & 2 & 2 \\ -1 & -1 & 2 \end{pmatrix}$, $T=\begin{pmatrix} -1 \\ 4 \\ -5 \end{pmatrix}$ by Gauss-seidal method .Suppose S is written in the form S=M-L-U, where M is a diagonal matrix is strictly lower triangular matrix U is strictly upper triangular matrix, if the iteration process is expressed as $X_{n+1} = QX_n + F$,then find Q.

(15)

(b) Consider the function f defined by$f(x, y) = x - [x]$, where x is the positive variable and $[x]$ is the integral part of x.Show that it is discontinuous for integral values of x and continuous for all other. Also draw its graph.

(15)

6. (a) Find the absolute that: $2sinA = |sin2|A$ **(15)**

(b) Test for convergence the following series

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \cdots$$

(15)

7 **(a)** A manufacturer, who produces medicine bottles, find that .1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug Manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution find how many boxes will contain:

1) No defective 2) at least two defective, bottles. **(10)**

(b) If x_1, x_2 and x_3 be uncorrelated variables each having the same standard deviation, obtain the correlation coefficient between

$$u = x_1 + x_2 \text{ and } v = x_2 + x_3$$ **(10)**

(c) Find the angle between the surfaces $x \log z = y^2 - 1$ and $x^2 y = 2 - z$ at the point $(1,1,1)$. **(10)**